This is PRACTICE for
CSC250: Theory of Computation
as taught by Pablo Frank-Bolton in Fall 2021.

The following materials are **permitted** while taking this examination:

- Cheatsheet

Honor code: no other resources are permitted during this exam.
This includes (but is not limited to): online materials, book, or class notes.

You may take **at most** one problem home as a take-home problem. To be solved with the exact same rules as with the exam. This question is due on 3:00PM EST Thursday October 28, 2021. You must upload a Typed (Word/Latex) pdf of the solution to Moodle (Midterm take-home question Submission Link)

YOUR NAME: _____

Question 1. Valid or Invalid Reasoning (4 points)

For each of the following English arguments, express the argument in terms of propositional logic and briefly justify whether the argument is valid or invalid. Be sure to clearly label your propositions.

(a) *When the weather is nice, Max either rides their bike on the rail trail or goes for a walk through the gardens (but never both on the same day). The weather is nice, and Max is going for a walk through the gardens. Therefore, Max will not ride their bike on the rail trail today.*

(b) *When students go to Neilson Library, they want to check out a book. No students went to Neilson Library today. This means that no one wanted to check out a book.*

Question 2. Interpreting regular expressions (6 points)

(a) Describe the language matched by the following regular expression:

$$0(10)^*1 + 1(01)^*0$$

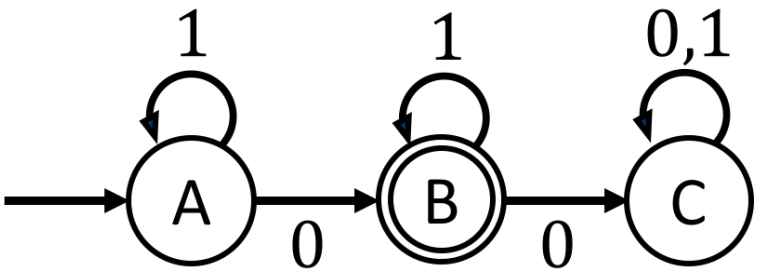
(b) Consider the following language on the alphabet $\Sigma = \{0, 1\}$:

$$L = \{w \mid w \texttt{ has even length, and starts and ends with the same symbol}\}$$

Write a regular expression for L.

Question 3. Interpreting Finite Automata (4 points)

Consider the following finite automaton:



(a) What is the start state?

(b) What is the set of accepting states?

(c) Is this a DFA, an NFA, neither, or both?

(d) What is the language accepted by this FA?

Question 4. Building Finite Automata (6 points)

Draw the transition diagram for a finite automaton that recognizes each of the following languages. In all cases, the alphabet is $\Sigma = \{0, 1\}$.

(a) $\{w \in \Sigma^* \mid w$ `begins with` 1 `and ends with` $0\}$.

(b) $\{w \in \Sigma^* \mid w$ `contains an even number of` $1s\}$.

(c) $\{w \in \Sigma^* \mid w$ `does not contain the substring` $10\}$.

Question 5. Short proofs (8 points)

Determine whether each of the following statements is `true` or `false`.
If it is `true`, provide a short proof. If it is `false`, give a counterexample.

(a) All regular languages are finite.

(b) All finite languages are regular.

(c) All finite automata accept the empty string ε.

(d) There exists some finite automaton that accepts the empty string ε.

Question 6. Non-Regular Languages (6 points)

Prove that the language:

$$DOUBLEZERO = \{w \mid w \texttt{ contains twice as many } 0s \texttt{ as 1s}\}$$

is NOT a regular language.

Question 7. Context-Free Languages (6 points)

Prove that the language:

$$DOUBLEZERO = \{w \mid w \texttt{ contains twice as many } 0s \texttt{ as 1s}\}$$

is context-free.

Question 8. Decidable Languages (2 points)

Prove that the language:

$$DOUBLEZERO = \{w \mid w \texttt{ contains twice as many } 0s \texttt{ as 1s}\}$$

is decidable.

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